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*Invited Paper, Special Issue on Inverse Problems, *Matematica Aplicada e Computacional.*

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I. INTRODUCTION

In this paper we present theoretical results for computational procedures developed to estimate damping in distributed parameter models of flexible structures. These efforts are part of continuing investigations of damping models for composite material structures that have been the focus of our attention for the past several years. Our interest in these models grew out of questions related to control and stabilization of such structures. To solve problems related to design, control and stabilization of large flexible structures, one requires a) an accurate mathematical model of the dynamics of the structure, and b) a method or algorithm for estimating the parameters in the model. Most structures, including the simplest of beams, exhibit some type of damping behavior, and it is important for the development of accurate models to account for the damping mechanism. Various models have been proposed for such damping mechanisms (see [CP] and [R]), each grounded upon reasonable physical principles and yet leading to very different mathematical models. It is important then, in the effort to develop accurate models for flexible structures, to study the various damping mechanisms and their implications for estimation, stabilization, and control problems. Recently, Banks, Inman et al ([BWIC]) reported on their study of Kelvin-Voigt and viscous damping in modeling and parameter estimation for composite beams. In related investigations Banks and Ito (see [BI]) developed a general framework for parameter estimation (including numerical convergence results) for a large class of beam models with damping (including viscous, Kelvin-Voigt, spatial-hysteresis). A notable exception that cannot be treated using results of these studies is viscoelastic damping of Boltzmann type (or "time hysteresis"). Our focus in this note will be on this type of damping. We discuss below theoretical and computational results for the estimation of parameters in a model (described in section II) of a beam with tip body and Boltzmann type damping. We illustrate use of the resulting procedures with data from experiments involving a composite material beam with attached tip body.

II. Mathematical Formulation

The mathematical model of interest to us is the following system for the transverse vibrations of a cantilevered Euler-Bernoulli beam with tip body and Boltzmann damping:

$$\rho u_{tt}(t,x) + \frac{\partial^2}{\partial x^2}\left\{EIu_{xx}(t,x) - \int_{-r}^{0} g(s)u_{xx}(t+s,x)\,ds\right\} = f(t,x) \tag{2.1}$$

$$0 < x < l, \qquad t > 0,$$

$$u(t,0) = \frac{\partial u}{\partial x}(t,0) = 0, \qquad t > 0,$$

$$mc\frac{\partial^2 u}{\partial t^2}(t,l) + (J + mc^2)\frac{\partial}{\partial x}\frac{\partial^2 u}{\partial t^2}(t,l) + \left\{EIu_{xx} - \int_{-r}^{0} g(s)u_{xx}(t+s,x)\,ds\right\}\Big|_{x=l} = k(t) \tag{2.2}$$

$$m\frac{\partial^2 u}{\partial t^2}(t,l) + mc\frac{\partial}{\partial x}\frac{\partial^2 u}{\partial t^2}(t,l) - \frac{\partial}{\partial x}\left\{EIu_{xx} - \int_{-r}^{0} g(s)\; u_{xx}(t+s,x)\,ds\right\}\Big|_{x=l} = h(t)$$

$$u(0,x) = u_t(0,x) = 0, \qquad 0 < x < l.$$

Here $u(t,x)$ is the transverse displacement at time t and position x along the beam. Also, ρ is the linear mass density, EI is the stiffness, and $f(t,x)$ represents external applied forces. The boundary conditions represent a beam clamped at $x = 0$ and free at $x = l$, with the tip body dynamics governed by (2.2). We assume that the tip body has mass m and moment of inertia J about its center of mass which is assumed to be located at a distance c from the tip of the beam along the beam's axis (see [BR] for a more detailed discussion of models for beams with tip bodies). The functions $k(t)$ and $h(t)$ denote externally applied moments (k) and transversally applied forces (h) exerted on the tip body. The delay length r (thus the terminology "time hysteresis") is positive, possibly infinite. In this paper we assume that the kernel $g(s)$ (the damping kernel) has the form

$$g(s) = \frac{\alpha e^{\beta s}}{(-s)^p} \qquad -r \le s < 0,$$

where α, β are positive constants, and p is fixed and satisfies $0 \le p < 1$.

Let $q = (EI, \alpha, \beta, \rho, m, J) \in Q \subset \mathbf{R}^6$ denote the parameters of interest, where the admissible parameter set Q is a compact subset of $\mathbf{R}^6$. (The theory can be modified to include spatially varying EI, but this is not our focus here; see [BR], [BC]). It should be noted that we fix values for r and p rather than treat them as parameters. It is also interesting to treat r and p as parameters, but this requires the introduction of more complicated parameter-dependent state spaces (see [BBC] for an example of the use of such state-spaces). We have carried out numerical studies in this direction but will defer any discussion on this aspect of these models to a subsequent paper.

Here we show (similar in spirit to the approaches in [BR], [BGRW]) how to reformulate (2.1) - (2.2) as an abstract Cauchy problem, and in this framework we pose the problem of estimating q as a least-squares fit to data. Having done this, we will in the next section discuss the construction of an approximation framework with which the estimation problem can be solved on a computer.

To proceed, let H denote the Hilbert space $\mathbf{R}^2 \times H^0(0,l)$ with inner product

$$\langle(\eta_1,\xi_1,\phi_1),(\eta_2,\xi_2,\phi_2)\rangle_H = \langle\Lambda(\eta_1,\xi_1),(\eta_2,\xi_2)\rangle_{\mathbf{R}^2} + \langle\phi_1,\phi_2\rangle_0,$$

where throughout we denote the standard $H^j(0,l)$ inner product by $\langle\cdot,\cdot\rangle_j$, and Λ is the 2×2 positive definite matrix

$$\Lambda = \frac{1}{\rho}\begin{bmatrix} m & mc \\ mc & J+mc^2 \end{bmatrix}.$$

Define the Hilbert space $V = \{(\eta,\xi,\phi) \in H : \ \phi \in H^2(0,l),\ \phi(0) = 0 = \phi'(0),\ \eta = \phi(l),\ \xi = \phi'(l)\}$. Denoting elements of V by $\Phi = (\phi(l),\phi'(l),\phi)$, $\Psi = (\psi(l),\psi'(l),\psi)$, we take as the V inner product $\langle\Phi,\Psi\rangle_V = \langle\phi,\psi\rangle_2$. It is clear that V is continuously and densely imbedded in H, and if we identify H with its dual H^* we have the pivot space framework $V \subset H = H^* \subset V^*$. Next, define the bilinear form σ on $V\times V$ by $\sigma(\Phi,\Psi) = \langle\phi'',\psi''\rangle_0$.

It is clear that the form σ is V-bounded and coercive. That is, there are constants $K>0,\ c_1>0$, so that for $\Phi,\Psi\in V$ we have

$$\left|\sigma(\Phi,\Psi)\right| \le K|\Phi|_V\,|\Psi|_V \tag{2.3}$$

and

$$\sigma(\Phi,\Phi) \ge c_1|\Phi|_V^2. \tag{2.4}$$

Now in the usual fashion (see [Kr], [Sh]) we can define an operator $A \in L(V, V^*)$ by

$$\langle A\Phi, \Psi \rangle_{V^*, V} = \sigma(\Phi, \Psi) \tag{2.5}$$

for all $\Phi, \Psi \in V$.

Next define the space

$$W = L_2(-r, 0; V; g_0),$$

with norm determined by the inner product

$$\langle \Gamma_1, \Gamma_2 \rangle_W = \int_{-r}^{0} g_0(s) \langle \Gamma_1(s), \Gamma_2(s) \rangle_V \, ds.$$

Here, $g_0(s) = \alpha_0 e^{\beta_0 s}/(-s)^p$, where α_0, β_0 are fixed values allowed for q in the admissible parameter set Q. As a notational convenience, for $q = (EI, \alpha, \beta, \rho, m, J) \in Q$, we mean by $g(q)(s)$ the function $g(q)(s) = \alpha e^{\beta s}/(-s)^p$. We point out that by virtue of the definition and the compactness of Q, the spaces $L_2(-r, 0; V; g(q))$ are equivalent for all $q \in Q$. Define the operator $D : \mathrm{dom} D \subset W \to W$ on the domain

$$\mathrm{dom} D = \left\{ \Gamma \in H^1(-r, 0; V; g_0) : \ \Gamma(0) = 0 \right\}$$

by $D\Gamma = d\Gamma/ds$. Next define the state space $Z = V \times H \times W$ with norm

$$\left| \begin{pmatrix} \Phi \\ \Psi \\ \Gamma \end{pmatrix} \right|_Z^2 = |\Phi|_V^2 + |\Psi|_H^2 + |\Gamma|_W^2 .$$

Also, define the parameter dependent state operator $\mathcal{A}(q)$ on the domain

$$\mathrm{dom} \mathcal{A}(q) = \left\{ \begin{pmatrix} \Phi \\ \Psi \\ \Gamma \end{pmatrix} \in Z : \begin{array}{c} \Psi \in V, \quad \Gamma \in \mathrm{dom} D \\ A(EI\Phi + \int_{-r}^{0} g(q)(s)(\Gamma(s) - \Phi) ds) \in H \end{array} \right\}$$

by

$$\mathcal{A}(q) \begin{pmatrix} \Phi \\ \Psi \\ \Gamma \end{pmatrix} = \begin{pmatrix} \Psi \\ -A(\frac{EI}{\rho}\Phi + \frac{1}{\rho}\int_{-r}^{0} g(q)(s)(\Gamma(s) - \Phi) ds) \\ \Psi + D\Gamma \end{pmatrix} . \tag{2.6}$$

Finally, if we define a state vector $z(t)$ by

$$z(t) = \begin{pmatrix} U(t) \\ \frac{d}{dt}U(t) \\ U(t) - U(t+\cdot) \end{pmatrix}$$

where $U(t) = (u(t,l), u_x(t,l), u(t,\cdot))$, then (2.1) - (2.2) can be reformulated as the following equivalent abstract Cauchy problem on Z:

$$\dot{z}(t) = \mathcal{A}(q)z(t) + F(t), \quad z(0) = z_0 \tag{2.7}$$

where $F(t) = \begin{pmatrix} 0 \\ \overline{F}(t) \\ 0 \end{pmatrix}$ with $\overline{F}(t) = \frac{1}{\rho}\left(\Lambda^{-1}\begin{pmatrix} h(t) \\ k(t) \end{pmatrix}, f(t,\cdot)\right) \in H$. Before giving a well-posedness result for (2.7), we make the following assumption on the admissible parameters.

A1) There exists $\epsilon > 0$ such that

$$\rho > \epsilon, \quad EI - \int_{-r}^{0} g(q)(s)ds \geq \epsilon \quad \text{for all } q \in Q.$$

This is a physically reasonable assumption (see [Wa]), basically guaranteeing that the structure is a solid. We shall have occasion to consider the space Z_q, which is the Hilbert space consisting of the elements of Z equipped with the inner product

$$\begin{aligned} \langle(\Phi_1, \Psi_1, \Gamma_1), (\Phi_2, \Psi_2, \Gamma_2)\rangle_q &= (EI - \int_{-r}^{0} g(q)(s)ds)\sigma(\Phi_1, \Phi_2) + \rho\langle\Psi_1, \Psi_2\rangle_H \\ &+ \int_{-r}^{0} g(q)(s)\sigma(\Gamma_1(s), \Gamma_2(s))ds, \end{aligned}$$

for $q = (EI, \alpha, \beta, \rho, m, J) \in Q$. Since Q is compact, one can readily see that these inner products are equivalent on Z for all $q \in Q$. Next we collect some results concerning the operators $\mathcal{A}(q)$ and D which we have defined.

Theorem 2.1.

i) Under assumption A1), the operator $\mathcal{A}(q)$ is dissipative in Z_q for each $q \in Q$. Further, $\mathcal{A}(q)$ generates a C_0-semigroup $T(q)(t)$ on Z which satisfies $|T(q)(t)|_Z \leq Me^{\omega t}$, where the constants M, ω are independent of $q \in Q$.

ii) The operator D is dissipative on W, and the resolvent $(\lambda I - D)^{-1}$ exists for $\lambda > 0$.

These results follow by extending the arguments found in [FI]. We can thus conclude that the system (2.7) is well-posed.

We may now formulate the estimation problem of interest as follows: Find $q \in Q$ that minimizes

$$\mathcal{J}(q) = \sum_i |v(q)(t_i, l) - \nu_i|^2. \tag{2.8}$$

The values ν_i are observations of the tip velocity of the beam (obtained, say, via a laser vibrometer) at time t_i, and $v(q)(t_i, l)$ is the fourth component of the state vector $z(q)(t)$ (i.e., the first component of $\frac{d}{dt}U(t)$), where $z(q)(t)$ is the mild solution of (2.7) given by

$$z(q)(t) = T(q)(t)z_0 + \int_0^t T(q)(t-s)F(s)ds. \tag{2.9}$$

That this inverse problem is well-posed will follow from the compactness of Q if we show that $\mathcal{J}$ is continuous in q. But in view of (2.9), under reasonable assumptions on the initial data z_0 and the forcing term $F(t)$, this follows if we show that $T(\tilde{q})(t)z \longrightarrow T(q)(t)z$ for arbitrary $\tilde{q} \to q$ and $z \in Z$. The following theorem provides the desired results.

Theorem 2.2. *If $\tilde{q} \to q$, then $T(\tilde{q})(t)z \longrightarrow T(q)(t)z$ for all $z \in Z$, uniformly in bounded t-intervals.*

Proof Since the generators $\mathcal{A}(q)$ are dissipative on Z_q, the result follows from the well known Trotter-Kato theorem (see [Pa]) once we show continuity in q of the resolvents $R_\lambda(\mathcal{A}(q))$. To this end, let

$$z = z(q) = \begin{pmatrix} \Phi \\ \Psi \\ \Gamma \end{pmatrix} = R_\lambda(\mathcal{A}(q))\,x \tag{2.10}$$

and

$$\tilde{z} = \tilde{z}(\tilde{q}) = \begin{pmatrix} \tilde{\Phi} \\ \tilde{\Psi} \\ \tilde{\Gamma} \end{pmatrix} = R_\lambda(\mathcal{A}(\tilde{q}))\,x \tag{2.11}$$

where

$$x = \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} \in Z.$$

We must show that, for some $\lambda > \omega$, $\tilde{z} \to z$ as $\tilde{q} \to q$. Fix $\lambda > \max\{\omega, 0\}$. From (2.10) we have

$$\lambda\Phi - \Psi = \xi$$

$$\lambda\Psi + \frac{1}{\rho}A\left(EI\Phi + \int_{-r}^{0} g(q)(s)(\Gamma(s) - \Phi)ds\right) = \eta$$

$$\lambda\Gamma - \Psi - D\Gamma = \zeta.$$

Substituting the first and third equations into the second, we have

$$\lambda^2\Phi + \frac{1}{\rho}\left(EI - \int_{-r}^{0} g(q)(s)e^{\lambda s}ds\right)A\Phi$$

$$= \eta + \lambda\xi - \frac{1}{\rho}\int_{-r}^{0} g(q)(s)A\left[(\lambda I - D)^{-1}(\zeta - \xi)\right]ds. \tag{2.12}$$

Similarly, from (2.11) we have

$$\lambda^2\tilde{\Phi} + \frac{1}{\tilde{\rho}}\left(\tilde{E}I - \int_{-r}^{0} g(\tilde{q})(s)e^{\lambda s}ds\right)A\tilde{\Phi}$$

$$= \eta + \lambda\xi - \frac{1}{\tilde{\rho}}\int_{-r}^{0} g(\tilde{q})(s)A\left[(\lambda I - D)^{-1}(\zeta - \xi)\right]ds. \tag{2.13}$$

Define the bilinear form σ_λ on $V \times V$ by

$$\sigma_\lambda(q)(\Phi, \Psi) = \lambda^2\langle\Phi, \Psi\rangle_H + \frac{1}{\rho}\left(EI - \int_{-r}^{0} g(q)(s)e^{\lambda s}ds\right)\sigma(\Phi, \Psi). \tag{2.14}$$

Since $\lambda > 0$, it is clear that σ_λ is V-coercive [this follows from A1) since $-\int_{-r}^{0} g(q)(s)ds \leq -\int_{-r}^{0} g(q)(s)e^{\lambda s}ds$]. That is, there exists $c > 0$ and $\hat{\lambda}$ so that

$$c|\Phi|_V^2 + \hat{\lambda}|\Phi|_H^2 \leq \sigma_\lambda(q)(\Phi, \Phi) \qquad \text{for all } q \in Q. \tag{2.15}$$

Next note that it follows from the form of $g(q)(s)$ that for $\mu \in H^1(-r, 0)$,

$$\int_{-r}^{0^-} |g(q)(s) - g(\tilde{q})(s)|\mu(s)ds \longrightarrow 0 \tag{2.16}$$

as $\tilde{q} \to q$. From this it follows that

$$\begin{aligned}
|\sigma_\lambda(q)(\Phi,\Psi) - \sigma_\lambda(\tilde{q})(\Phi,\Psi)| &\leq \left|\frac{\tilde{EI}}{\tilde{\rho}} - \frac{EI}{\rho}\right| |\sigma(\Phi,\Psi)| \\
&\quad + \int_{-r}^{0} \left|\frac{1}{\rho} g(q)(s) - \frac{1}{\tilde{\rho}} g(\tilde{q})(s)\right| e^{\lambda s} ds \cdot K |\Phi|_V |\Psi|_V \\
&\leq d_\lambda(q,\tilde{q}) \cdot |\Phi|_V \cdot |\Psi|_V \qquad (2.17)
\end{aligned}$$

where $d_\lambda(q,\tilde{q}) \to 0$ as $\tilde{q} \to q$.

Now, since $\Psi - \tilde{\Psi} = \lambda(\Phi - \tilde{\Phi})$, and $\Gamma - \tilde{\Gamma} = (1 - e^{\lambda s})(\Phi - \tilde{\Phi})$, we will be finished with the proof if we show that $|\Phi - \tilde{\Phi}|_V \to 0$ as $\tilde{q} \to q$. To do this, we make use of the above-defined coercive form σ_λ. From (2.15) and (2.12) - (2.13), we have

$$\begin{aligned}
c|\Phi - \tilde{\Phi}|_V^2 + \hat{\lambda}|\Phi - \tilde{\Phi}|_H^2 &\leq \sigma_\lambda(\tilde{q})(\Phi - \tilde{\Phi}, \Phi - \tilde{\Phi}) \\
&= \left[\sigma_\lambda(\tilde{q})(\Phi, \Phi - \tilde{\Phi}) - \sigma_\lambda(q)(\Phi, \Phi - \tilde{\Phi})\right] \\
&\quad + \left[\sigma_\lambda(q)(\Phi, \Phi - \tilde{\Phi}) - \sigma_\lambda(\tilde{q})(\tilde{\Phi}, \Phi - \tilde{\Phi})\right] \\
&\leq d_\lambda(q,\tilde{q})|\Phi|_V |\Phi - \tilde{\Phi}|_V \\
&\quad + K \int_{-r}^{0} \left|\frac{1}{\rho} g(q)(s) - \frac{1}{\tilde{\rho}} g(\tilde{q})(s)\right| \cdot \left|\int_s^0 e^{\lambda(s-\theta)}[\zeta(\theta) - \xi] d\theta\right|_V ds |\Phi - \tilde{\Phi}|_V .
\end{aligned}$$

Cancelling a term $|\Phi - \tilde{\Phi}|_V$ from each side of the inequality, we obtain an estimate from which the result follows.

Let us make a few remarks about the proof. Many proofs of convergence of a sequence of semigroups involve the application of some version of the Trotter-Kato theorem. This typically involves showing convergence either of the generators of the semigroups or of their resolvents, depending upon the version of the Trotter-Kato theorem which is used. For parameter estimation problems such as we are considering here, the "resolvent convergence" form of the Trotter-Kato theorem is preferable as it leads to convergence arguments similar in spirit to those of the "variational approach" of finite element theory (i.e., the definition and use of the form σ_λ). Also, for problems involving spatially varying parameters, this version allows one to impose weaker smoothness requirements on the parameters in order to prove convergence results (for a more detailed discussion, see [B], [BI]).

These remarks will be relevant later in the paper when, after constructing an approximation scheme for our estimation problem, we again apply the Trotter-Kato theorem to argue convergence of a sequence of approximating semigroups.

III. Approximation Framework

Constructing a finite dimensional approximation scheme for the parameter estimation problem (2.8) - (2.9) involves approximating the state operator $\mathcal{A}(q)$ and the state space Z. (Were Q also infinite dimensional, which is often the case for estimation problems, we would also need finite dimensional spaces Q^K to approximate Q. Treatment of this case is possible but we will not pursue it here.) We observe that approximating $\mathcal{A}(q)$ and Z involves two stages - approximation in the spatial variable, and approximation in the delay variable. We will see that the two approximation stages are independent, so that our task in constructing a scheme for (2.8) - (2.9) reduces to choosing a reasonable spatial approximation (such as finite elements) and a reasonable delay approximation scheme (there are several in the literature (see [BB], [BK], [IK])). That is, there is no conditional relationship between the spatial and delay variable approximations. For definiteness, we use the index N for the spatial variable, and M for the delay variable.

Let us consider first the approximation in the spatial variable. This amounts to choosing a sequence of finite dimensional spaces H^N satisfying $H^N \subset V \subset H$ which approximate the spaces V and H (see A2) below). Once the H^N are given, the rest of the approximation in the spatial variable follows naturally. That is, the form σ defines operators $A^N : H^N \to H^N$ by

$$\langle A^N \Phi, \Psi \rangle_H = \sigma(\Phi, \Psi) \qquad \text{for all } \Phi, \Psi \in H^N.$$

Also, define spaces $W^N \subset W$ by $W^N = L_2(-r, 0; H^N; g_0)$ and $Z^N \subset Z$ by $Z^N = H^N \times H^N \times W^N$. Let $P_H^N : H \to H^N$, $P_V^N : V \to H^N$, and $P_W^N : W \to W^N$ denote the respective orthogonal projections, and define the operator D^N on the domain

$$\text{dom} D^N = (\text{dom} D) \cap W^N$$

by

$$D^N\Gamma = D\Gamma.$$

Finally, define the state operator $\mathcal{A}^N(q)$ on the domain

$$\mathrm{dom}\mathcal{A}^N(q) = \left\{ \begin{pmatrix} \Phi \\ \Psi \\ \Gamma \end{pmatrix} \in Z^N : \quad \Gamma \in \mathrm{dom}D^N \right\}$$

by

$$\mathcal{A}^N(q) \begin{pmatrix} \Phi \\ \Psi \\ \Gamma \end{pmatrix} = \begin{pmatrix} \Psi \\ -\frac{EI}{\rho} A^N\Phi - \frac{1}{\rho}\int_{-r}^{0} g(q)(s) A^N(\Gamma(s) - \Phi) ds \\ \Psi + D^N\Gamma \end{pmatrix}. \tag{3.1}$$

We note that each $\mathcal{A}^N(q)$ generates a C_0-semigroup on Z^N (this involves a slight modification of the proof that $\mathcal{A}(q)$ generates a C_0-semigroup).

Next, we consider approximation in the delay variable. That is, we apply one of several schemes in the literature for delay equations to the operators $\mathcal{A}^N(q)$ and spaces Z^N. This amounts to defining (for each N) finite dimensional subspaces $W^{N,M} \subset W^N$ and operators $D^{N,M} : W^{N,M} \longrightarrow W^{N,M}$. However, we are interested in imposing conditions on the delay variable approximation which are independent of the spatial approximation (hence independent of N). In order to do this, we will consider the generic space $\bar{W} = L_2(-r, 0; \mathbf{R}; g_0)$, where $\mathbf{R} = \mathbf{R}^1$, and the generic operator $\bar{D}$ defined on the domain

$$\mathrm{dom}\bar{D} = \left\{ w \in H^1(-r, 0; \mathbf{R}; g_0) : \quad w(0) = 0 \right\}$$

by

$$\bar{D}w = \frac{dw}{ds}.$$

A delay approximation scheme consists of finite dimensional subspaces $\bar{W}^M \subset \bar{W}$ and operators $\bar{D}^M : \bar{W}^M \longrightarrow \bar{W}^M$ satisfying certain conditions (see A3) and A4) below) which are independent of N. However, although $\bar{W}^M$ and $\bar{D}^M$ are independent of N, for each N they define in a natural way subspaces $W^{N,M} \subset W^N$ and operators $D^{N,M} : W^{N,M} \longrightarrow W^{N,M}$. That is, suppose

$$\bar{W}^M = \left\{ \sum_j w_j e_j^M : \ w_j \in \mathbf{R} \right\},$$

where ϵ_j^M are given basis functions for $\bar{W}^M$. Then define $W^{N,M}$ by

$$W^{N,M} = \left\{ \sum_j \Gamma_j \epsilon_j^M \,:\, \Gamma_j \in H^N \right\}. \tag{3.2}$$

If $H^N = \left\{ \sum_i a_i B_i^N : a_i \in \mathbf{R} \right\}$, where the B_i^N are basis functions for H^N, then any element Γ of $W^{N,M}$ can be written as

$$\Gamma = \sum_j \Gamma_j \epsilon_j^M = \sum_j \left(\sum_i a_i^j B_i^N \right) \epsilon_j^M, \quad a_i^j \in \mathbf{R}.$$

Hence we define $D^{N,M}$ by

$$D^{N,M}\left(\sum_j \left(\sum_i a_i^j B_i^N \right) \epsilon_j^M \right) = \sum_i \left(\bar{D}^M (\textstyle\sum_j a_i^j \epsilon_j^M) \right) B_i^N. \tag{3.3}$$

In a similar manner, the orthogonal projection $\bar{\Pi}^M : \bar{W} \to \bar{W}^M$ defines naturally the orthogonal projection $\Pi^{N,M} : W^N \to W^{N,M}$. This completes the approximation of the delay variable.

Having the approximation of the spatial variable (the spaces H^N) and the delay variable (the spaces $\bar{W}^M$ and operators $\bar{D}^M$), we define the finite dimensional state space $Z^{N,M}$ by

$$Z^{N,M} = H^N \times H^N \times W^{N,M} \tag{3.4}$$

and state operator $\mathcal{A}^{N,M}(q)$ on $Z^{N,M}$ by

$$\mathcal{A}^{N,M}(q) \begin{pmatrix} \Phi \\ \Psi \\ \Gamma \end{pmatrix} = \begin{pmatrix} \Psi \\ -\frac{EI}{\rho} A^N \Phi - \frac{1}{\rho} \int_{-r}^0 g(q)(s) A^N(\Gamma(s) - \Phi) ds \\ \Psi + D^{N,M}\Gamma \end{pmatrix}. \tag{3.5}$$

Finally, we state the finite dimensional estimation problem of interest: Find $q \in Q$ that minimizes

$$\mathcal{J}^{N,M}(q) = \sum_i |v^{N,M}(q)(t_i, l) - \nu_i|^2 \tag{3.6}$$

where $v^{N,M}(q)(t,l)$ is the fourth component of the state vector $z^{N,M}(q)(t)$ given by

$$z^{N,M}(q)(t) = T^{N,M}(q)(t) P_Z^{N,M} z_0 + \int_0^t T^{N,M}(q)(t-s) P_Z^{N,M} F(s) ds. \tag{3.7}$$

Here $T^{N,M}(q)$ is the semigroup generated by $\mathcal{A}^{N,M}(q)$ and $P_Z^{N,M}$ is the canonical orthogonal projection of Z onto $Z^{N,M}$. Using arguments similar to those in the proof of Theorem 2.2, it is readily seen that $Z^{N,M}$ and $\mathcal{J}^{N,M}$ are continuous in q for q in the compact set Q. Thus, for each N, M, the problem for (3.6) - (3.7) has a solution $\bar{q}^{N,M}$. To obtain convergence and continuous dependence (with respect to the observations ν_i) of the parameter estimates $\bar{q}^{N,M}$ obtained in the finite dimensional problems for (3.6), it suffices under the assumption that Q is compact (for details, see [B], [BKu]) to argue: For arbitrary $\{q^{N,M}\} \subset Q$ with $q^{N,M} \to q$, we have $z^{N,M}(q^{N,M})(t) \to z(q)(t)$ where $z^{N,M}$ and z are given by (3.7) and (2.9), respectively. It is readily argued that this convergence holds under reasonable assumptions on z_0 and $F(t)$ if one first argues that $T^{N,M}(q^{N,M})(t)P_Z^{N,M}z \to T(q)(t)z$ for arbitrary $q^{N,M} \to q$ in Q and $z \in Z$. We give the arguments for this latter convergence in the next section.

IV. Convergence Analysis

To give the desired convergence arguments, we need several approximation assumptions, which we now state.

A2) $H^N \subset V \subset H$ satisfies $\left|P_H^N\Phi - \Phi\right|_V \to 0$ and $\left|P_V^N\Phi - \Phi\right|_V \to 0$ as $N \to \infty$.

A3) $\bar{W}^M \subset \bar{W}$ satisfies $\left|\bar{\Pi}^M w - w\right|_W \to 0$ as $M \to \infty$.

A4) $\bar{D}^M : \bar{W}^M \to \bar{W}^M$ satisfies

i) $\bar{D}^M$ is dissipative on $\bar{W}^M$ (uniformly in M),

ii) $(\lambda I - \bar{D}^M)^{-1}\bar{\Pi}^M w \longrightarrow (\lambda I - \bar{D})^{-1}w$ as $M \to \infty$ for $\mathrm{Re}\,\lambda > 0$.

Remark 4.1 Recall that to argue V-coerciveness of the form σ_λ, we used the fact that $\int_{-r}^0 g(s)e^{\lambda s}ds \le \int_{-r}^0 g(s)ds$. The function $e^{\lambda s}$ may be characterized by

$$e^{\lambda s} = \hat{f}(s) - \lambda(\lambda I - \bar{D})^{-1}\hat{f}(s),$$

where $\hat{f}(s) \equiv 1$. The delay approximation scheme defines a natural approximation of $e^{\lambda s}$ given by

$$e_\lambda^M(s) \equiv \bar{\Pi}^M\hat{f} - \lambda(\lambda I - \bar{D}^M)^{-1}\bar{\Pi}^M\hat{f}.$$

In order to argue V-coerciveness of a form σ_λ^M which we define below, we need $e_\lambda^M(s)$ to satisfy an inequality analogous to that given above:

$$\int_{-r}^{0} g(s)e_\lambda^M(s)ds \leq \int_{-r}^{0} g(s)ds \tag{4.1}$$

for all sufficiently large M.

Before proving the main semigroup convergence result of this section, we state the following lemma.

Lemma 4.1. *Assume that A3) and A4) hold, and that $W^{N,M}$ and $D^{N,M}$ are given by (3.2) and (3.3). Then $D^{N,M}$ is dissipative on $W^{N,M}$ (uniformly in N, M) and*

$$\left|(\lambda I - D^{N,M})^{-1}\Pi^{N,M}\Gamma - (\lambda I - D^{N})^{-1}\Gamma\right|_{W} \to 0 \tag{4.2}$$

as $M \to \infty$ for each N and each $\Gamma \in W^N$.

The proof is straightforward. We now prove the semigroup convergence result referred to above.

Theorem 4.2. *Assume that A1) – A4) and (4.1) hold. If $q^{N,M} \to q$ in Q, then $T^{N,M}(q^{N,M})P_Z^{N,M}z \to T(q)(t)z$ for all $z \in Z$, uniformly in bounded t-intervals.*

Proof First, the dissipativeness of $D^{N,M}$ implies that the $\mathcal{A}^{N,M}(q)$ are dissipative on Z_q, uniformly in N and M. Thus, we can apply the "resolvent convergence" form of the Trotter-Kato theorem, as in the proof of Theorem 2.2. Let

$$z = \begin{pmatrix} \Phi \\ \Psi \\ \Gamma \end{pmatrix} = R_\lambda(\mathcal{A}(q))x \tag{4.3}$$

and

$$z^{N,M} = \begin{pmatrix} \Phi^N \\ \Psi^N \\ \Gamma^{N,M} \end{pmatrix} = R_\lambda(\mathcal{A}^{N,M}(q^{N,M}))P_Z^{N,M}x, \tag{4.4}$$

where

$$x = \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} \text{ and } P_Z^{N,M}x = \begin{pmatrix} P_V^N\xi \\ P_H^N\eta \\ P_W^{N,M}\zeta \end{pmatrix} = \begin{pmatrix} \xi^N \\ \eta^N \\ \zeta^{N,M} \end{pmatrix}.$$

Solving (4.3) and (4.4) for Φ, Φ^N we find

$$\lambda^2\Phi + \frac{1}{\rho}\left(EI - \int_{-r}^{0} g(q)(s)e^{\lambda s}ds\right)A\Phi$$
$$= \eta + \lambda\xi - \frac{1}{\rho}\int_{-r}^{0} g(q)(s)A\left[(\lambda I - D)^{-1}(\zeta - \xi)\right]ds \tag{4.5}$$

and

$$\lambda^2\Phi^N + \frac{1}{\rho^{N,M}}\left(EI^{N,M} - \int_{-r}^{0} g(q^{N,M})(s)e_\lambda^M(s)ds\right)A^N\Phi^N$$
$$= \eta^N + \lambda\xi^N - \frac{1}{\rho^{N,M}}\int_{-r}^{0} g(q^{N,M})(s)A^N\left[(\lambda I - D^{N,M})^{-1}(\zeta^{N,M} - \xi^N)\right]ds. \tag{4.6}$$

As in the proof of Theorem 2.2, it is sufficient to argue that $|\Phi^N - \Phi|_V \to 0$ as $q^{N,M} \to q$. To this end, define a bilinear form σ_λ^M on $V \times V$ by

$$\sigma_\lambda^M(q)(\Phi, \Psi) = \lambda^2\langle\Phi, \Psi\rangle_H + \frac{1}{\rho}\left(EI - \int_{-r}^{0} g(q)(s)e_\lambda^M(s)ds\right)\sigma(\Phi, \Psi). \tag{4.7}$$

From the technical condition (4.1) it follows that σ_λ^M is V-coercive, uniformly in M and q, for some $\lambda > 0$. That is, there exists $\bar{C} > 0$ so that

$$\bar{C}|\Phi|_V^2 + \hat{\lambda}|\Phi|_H^2 \leq \sigma_\lambda^M(q)(\Phi, \Phi) \tag{4.8}$$

for all $q \in Q$. Also, it is easy to see that for all $\Phi, \Psi \in V$,

$$\begin{aligned}|\sigma_\lambda(q)(\Phi, \Psi) - \sigma_\lambda^M(q)(\Phi, \Psi)| &\leq \left|\int_{-r}^{0} g(q)(s)\left[e^{\lambda s} - e_\lambda^M(s)\right]ds\right| \cdot K \cdot |\Phi|_V|\Psi|_V \\ &\leq K(M)|\Phi|_V|\Psi|_V\end{aligned} \tag{4.9}$$

where $K(M) \to 0$ as $M \to \infty$. Now, let $\tilde{\Phi}^N \in H^N$ satisfy $|\tilde{\Phi}^N - \Phi|_V \to 0$ as $N \to \infty$ (existence of a $\tilde{\Phi}^N$ follows from A2)). Thus it is sufficient to argue that $|\Phi^N - \tilde{\Phi}^N|_V \to 0$. Letting $\delta^N = \Phi^N - \tilde{\Phi}^N$, we have

$$\begin{aligned}&\bar{C}|\Phi^N - \tilde{\Phi}^N|_V^2 + \hat{\lambda}|\Phi^N - \tilde{\Phi}^N|_H^2 \leq \sigma_\lambda^M(q^{N,M})(\Phi^N - \tilde{\Phi}^N, \delta^N) \\ &= \sigma_\lambda^M(q^{N,M})(\Phi^N, \delta^N) - \sigma_\lambda(q)(\Phi, \delta^N) + \sigma_\lambda(q)(\Phi, \delta^N) - \sigma_\lambda^M(q^{N,M})(\tilde{\Phi}^N, \delta^N) \\ &= \langle\eta^N - \eta, \delta^N\rangle_H + \lambda\langle\xi^N - \xi, \delta^N\rangle_H \\ &\quad - \frac{1}{\rho^{N,M}}\int_{-r}^{0} g(q^{N,M})(s)\sigma\left((\lambda I - D^{N,M})^{-1}(\zeta^{N,M} - \xi^N), \delta^N\right)ds\end{aligned}$$

$$+\frac{1}{\rho}\int_{-r}^{0} g(q)(s)\sigma\left((\lambda I-D)^{-1}(\zeta-\xi),\delta^N\right)ds+\sigma_\lambda(q)(\Phi,\delta^N)-\sigma_\lambda^M(q^{N,M})(\tilde{\Phi}^N,\delta^N)$$

$$\leq c_2|\eta^N-\eta|_H|\delta^N|_V+c_3|\xi^N-\xi|_H\cdot|\delta^N|_V \qquad \text{(a)}$$

$$+\frac{K}{\rho^{N,M}}\int_{-r}^{0} g(q^{N,M})(s)|(\lambda I-D^{N,M})^{-1}(\zeta^{N,M}-\xi^N)-(\lambda I-D)^{-1}(\zeta-\xi)|_V ds|\delta^N|_V \qquad \text{(b)}$$

$$+\int_{-r}^{0}|\frac{1}{\rho^{N,M}}g(q^{N,M})(s)-\frac{1}{\rho}g(q)(s)|\int_{s}^{0}e^{\lambda(s-\theta)}|\zeta(\theta)-\xi|_V\,d\theta ds\cdot K\cdot|\delta^N|_V \qquad \text{(c)}$$

$$+\sigma_\lambda(q)(\Phi,\delta^N)-\sigma_\lambda(q^{N,M})(\Phi,\delta^N) \qquad \text{(d)}$$

$$+\sigma_\lambda(q^{N,M})(\Phi,\delta^N)-\sigma_\lambda^M(q^{N,M})(\Phi,\delta^N) \qquad \text{(e)}$$

$$+\sigma_\lambda^M(q^{N,M})(\Phi-\tilde{\Phi}^N,\delta^N) \qquad \text{(f)}$$

The proof is completed by applying A2) to (a), (4.2) to (b), (2.16) to (c), (2.17) to (d), (4.9) to (e), and the uniform V-boundedness of σ_λ^M to (f).

V. Computational Results

In this section we describe some of our computational results for the Boltzmann damping model under consideration. Recall that for the approximation scheme which we have described, the underlying spatial and delay discretizations should satisfy assumptions A2) – A4) and (4.1). For most of the numerical experiments which we have run, we have based our approximations on cubic splines for the spatial variable and a version of the so-called averaging scheme for the delay variable.

More specifically, for a beam of length l and a given positive integer N, consider the partition of the spatial interval $[0,l]$ given by $\Delta^N=\{x_i\}_{i=0}^N$, where $x_i=\frac{i}{N}\cdot l$. Let $S^3(\Delta^N)$ denote the set of cubic splines with knots Δ^N, and $\{\hat{b}_i^N\}_{i=-1}^{N+1}$ the standard cubic B-spline basis set for $S^3(\Delta^N)$ (see [Pr]). Next, we can in the usual way define a new set of functions $\{b_i^N\}_{i=1}^{N+1}$ so that the fixed end $(x=0)$ boundary conditions are satisfied. That is, define $b_1^N=2\hat{b}_{-1}^N-\hat{b}_0^N+2\hat{b}_1^N$, and $b_i^N=\hat{b}_i^N$ for $i=2,3,\dots,N+1$. Then define $B_i^N=(b_i^N(l),\frac{d}{dx}b_i^N(l),b_i^N)$, and set $H^N=\text{span}\,\{B_1^N,\dots,B_{N+1}^N\}$. For this choice of H^N, assumption A2) is satisfied.

For the delay variable, we use the "non-uniform mesh" version of the averaging scheme (see [FI]). That is, for a delay length r, positive integer M and kernel function $g(s)$, set $C = \int_{-r}^{0} g(s)ds$. Then the partition

$$-r = t_M^M < t_{M-1}^M < \cdots < t_1^M < t_0^M = 0$$

is defined uniquely by the condition $\int_{t_j^M}^{t_{j-1}^M} g(s)ds = \frac{C}{M}$. For the kernel functions $g(s)$ under consideration, which are increasing and have a singularity at $s = 0$, this results in a finer mesh near $s = 0$. Next, let χ_j^M denote the characteristic function on the interval $[t_j^M, t_{j-1}^M]$. Define the space $\bar{W}^M = \text{span}\ \{\chi_1^M, \ldots, \chi_M^M\}$. Finally, the operator $\bar{D}^M : \bar{W}^M \to \bar{W}^M$ is defined by

$$\bar{D}^M(\sum_{j=1}^{M} a_j \chi_j^M) = \sum_{j=1}^{M} \frac{a_{j-1} - a_j}{t_{j-1}^M - t_j^M} \chi_j^M,$$

where $a_0 \equiv 0$. It can be shown (see [FI]) that $\bar{W}^M$ and $\bar{D}^M$ satisfy A3)–A4) and that (4.1) is satisfied.

We have conducted a number of numerical tests using simulated data in which we observed the convergence and efficiency of the scheme for identifying the parameters EI, α, β, ρ, m and J. However, here we present an application using experimental data in order to demonstrate that the Boltzmann damping model can provide a very good prediction of vibration response in certain flexible structures. In particular, we use data from a series of experiments carried out at the Mechanical Systems Laboratory at SUNY at Buffalo. (We gratefully acknowledge Dr. D.J. Inman, H. Cudney, and Z. Liang for their collaboration on these experiments.) In these experiments, a cantilevered composite beam with tip body was excited from an initial rest configuration via an impulse from a force hammer. A laser vibrometer was used to collect measurements of the tip velocity of the beam in response to the force. These measurements were used as observations in our least-squares criterion $\mathcal{J}$ (recall (2.8)).

For the particular experiment presented here, we used a composite beam of length $l = 1.968$ ft., width 0.3345 ft., and thickness 0.01968 ft., with a tip body (a cylinder of radius 0.08036 ft. and height 0.27224 ft.) with mass $m = 0.075714$ slug and moment of inertia J given by $J = c_j^2 m$ with $c_j = 0.088265$ ft. The beam density was $\rho = 0.021441$

slug/ft. We observed the tip velocity of the beam after it was hit with an impulse hammer at $x_p = 0.984$ ft. The shape, height and duration of the input were modelled using the input signal from a transducer in the hammer. We recorded 2048 time observations at $\Delta t = 0.001953$ seconds intervals, thus giving time domain data over the period $[0, 4]$ seconds.

The actual structure (a composite material beam with a cylindrical body attached <u>near</u> the free end) used in our experiments entailed the "tip" body being attached with its center of mass at 1.9352 ft. from the clamped end of the beam of 1.968 ft. in length. So in fact we did not have a "tip body" as described in the model above. To compensate for this inaccuracy in the model, we took $c = 0$ but allowed c_j and m (i.e., J and m) along with EI, α, and β to be estimated by our algorithm. We would expect to (and did) obtain "effective" values of c_j and m that are slight perturbations of the measured values given above.

We then used data from the experiment (1018 observations in the time period [0.01172,2] - we did not use any observations from the time period for the hammer hit) with the procedure we have described in order to estimate the stiffness parameter EI, the damping parameters α and β, and c_j and m. In these computations we chose $p = 0.5$ and $r = 0.01$. We obtained the values $EI = 103.892 \text{ lb} \cdot \text{ft}^2$, $\alpha = 48.469$, $\beta = 654.109$, $c_j = 0.091026$ ft., and $m = 0.070044$ slug. A value of EI was also determined, using standard modal techniques, to be $107.103 \text{ lb} \cdot \text{ft}^2$. Thus our value is well within the 10% range which is generally thought to be the accuracy of the modal techniques. The estimated values for c_j and m are also within 10% of the measured values. We point out that an heuristic physical interpretation of the large value of β is that the material has a "memory" which fades very rapidly. This type of short "memory" makes sense for materials which are "more elastic than viscous" as our experimental beam appeared to be. In fact, for the case of no "memory" at all (i.e. $g(s) = 0$), our model reduces to that of a purely elastic Euler-Bernoulli beam. Thus the presence of a small delay (the memory) in the mathematical model drastically changes the qualitative behavior of the model (from elastic to viscoelastic). (That small delays can affect behavior of a model has been observed elsewhere; see [DLP]).

We depict in Figures 1, 1a, and 2 the time domain response and FFT for our beam model and experiment. In these figures, the solid line graphs represent the experimental data while the dashed lines represent the model response corresponding to our estimated parameter values. It is clear that we have obtained a very good fit to data with this model. Other similar experiments and computational studies ([BWIC], [BFWIC]) illustrate the usefulness of the techniques described in this paper in studying damping in distributed parameter models for structures.

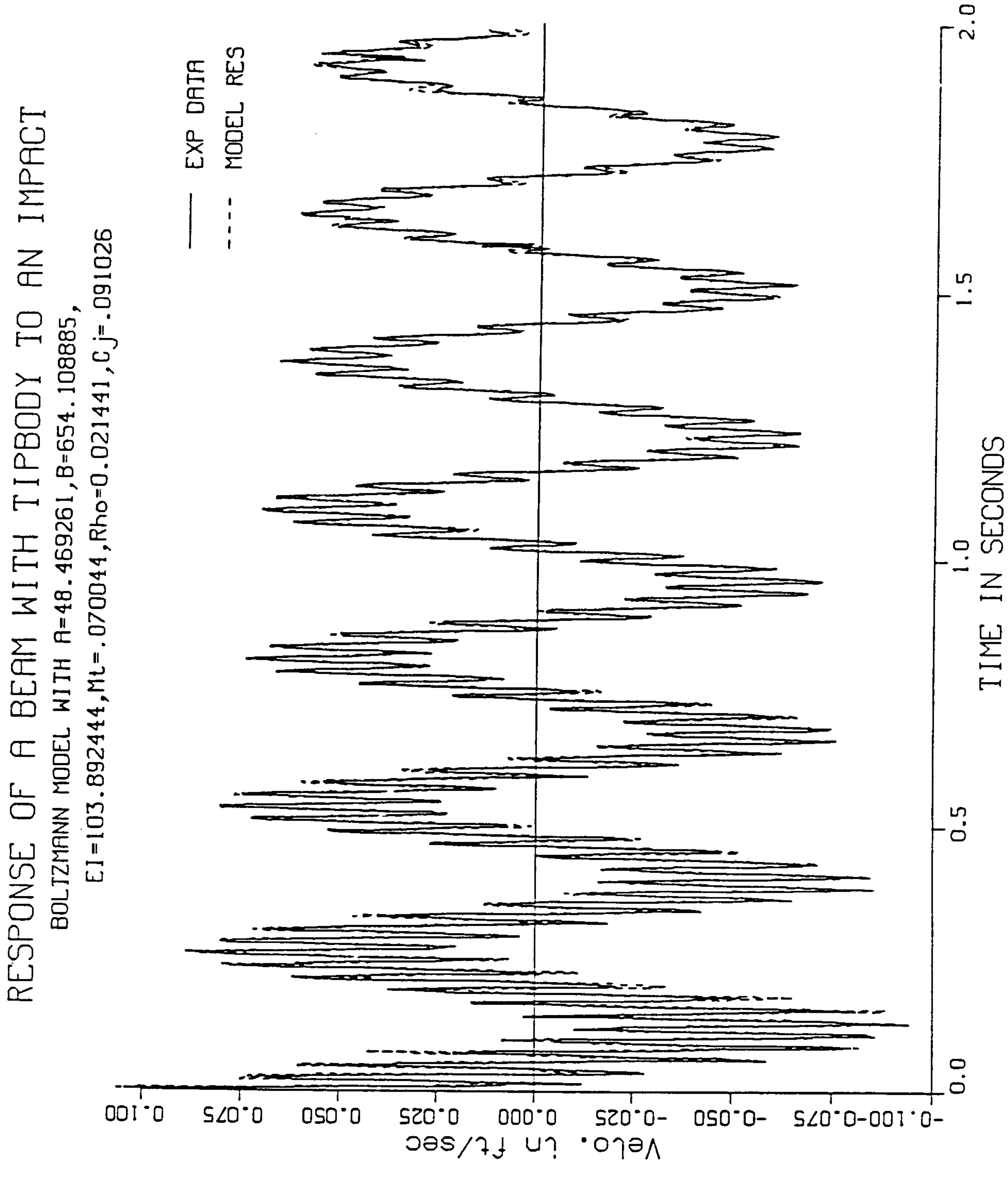
RESPONSE OF A BEAM WITH TIPBODY TO AN IMPACT
BOLTZMANN MODEL WITH A=48.469261,B=654.108885,
EI=103.892444,Mt=.070044,Rho=0.021441,Cj=.091026
EXP DATA
MODEL RES
TIME IN SECONDS
0.0
0.5
1.0
1.5
2.0
Velo. in ft/sec
0.100
0.075
0.050
0.025
0.000
-0.025
-0.050
-0.075
-0.100

Figure 1

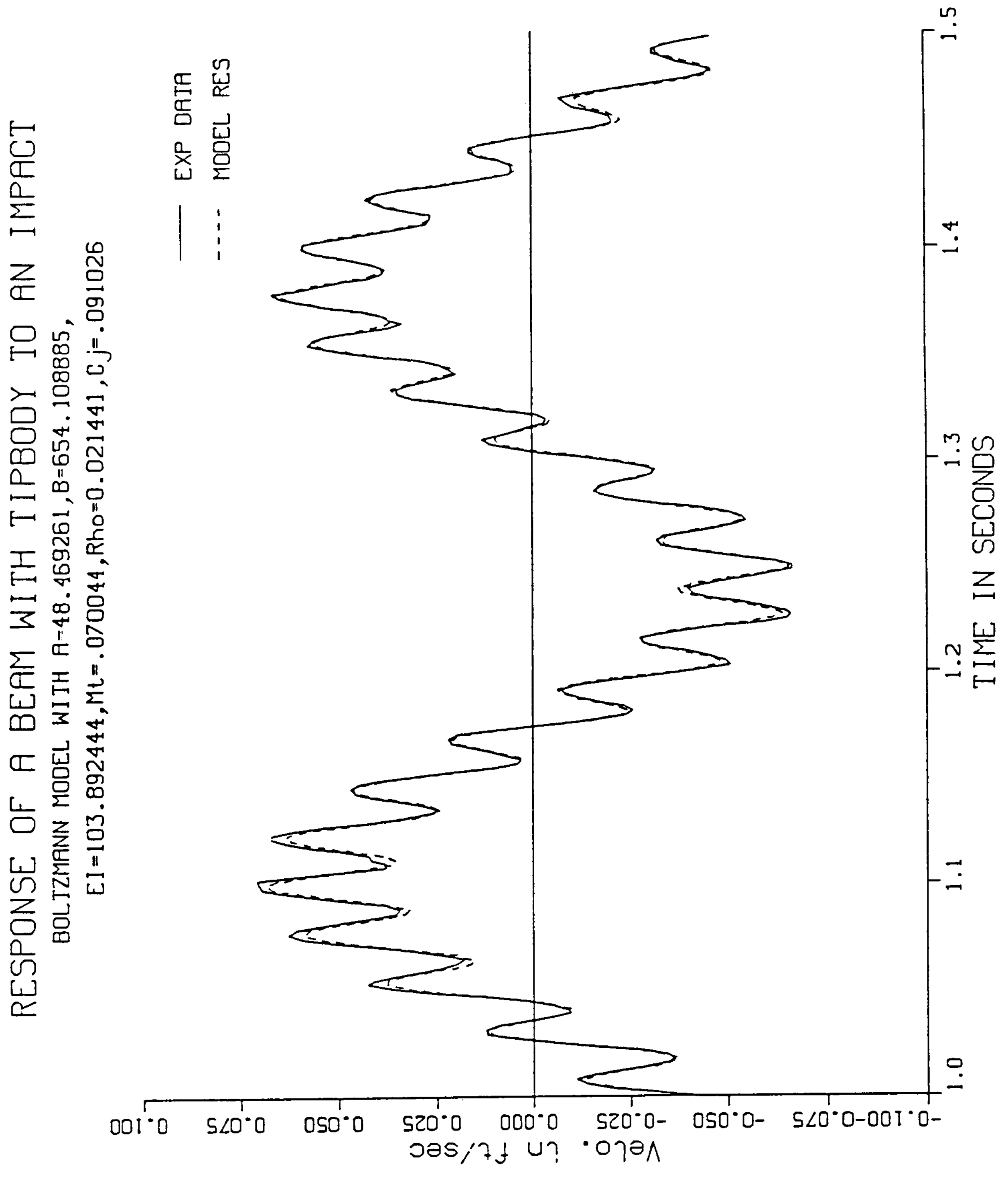
RESPONSE OF A BEAM WITH TIPBODY TO AN IMPACT
BOLTZMANN MODEL WITH A=48.469261,B=654.108885,
EI=103.892444,Mt=.070044,Rho=0.021441,Cj=.091026
EXP DATA
MODEL RES
TIME IN SECONDS
1.0
1.1
1.2
1.3
1.4
1.5
Velo. in ft/sec
0.100
0.075
0.050
0.025
0.000
-0.025
-0.050
-0.075
-0.100

Figure 1a

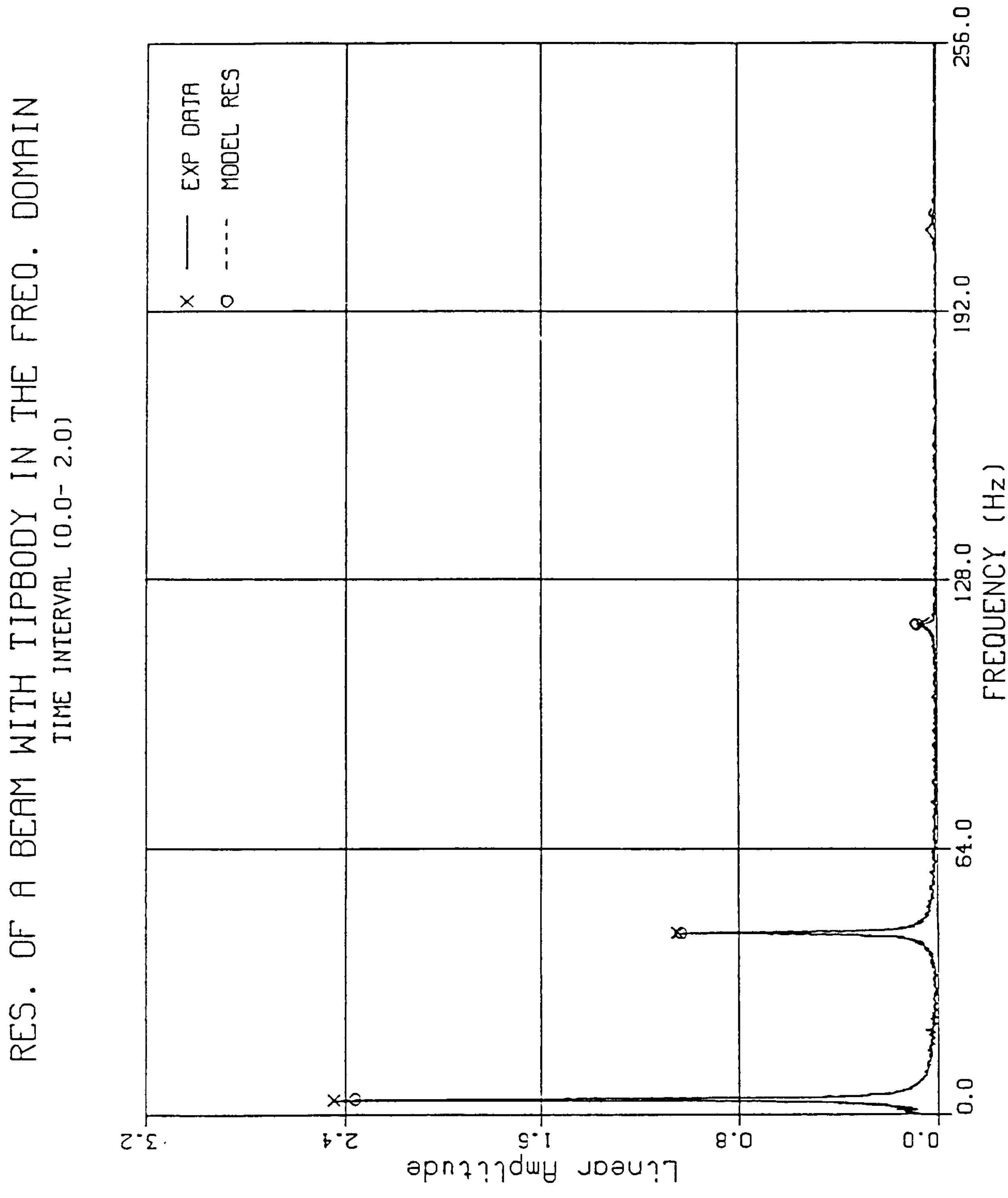
RES. OF A BEAM WITH TIPBODY IN THE FREQ. DOMAIN
TIME INTERVAL (0.0- 2.0)
× —— EXP DATA
o ---- MODEL RES
0.0
64.0
128.0
192.0
256.0
FREQUENCY (Hz)
0.0
0.8
1.6
2.4
3.2
Linear Amplitude

Figure 2

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Final Report

16. Abstract

We present a model for a flexible beam with time hysteresis (Blotzmann type viscoelasticity) damping and tip body. A computational method for the estimation of the damping parameters is developed, and theoretical convergence/continuous dependence results are given. An example is presented in which experimental data is used, demonstrating the efficacy of the computational method and the plausibility of the model for predicting response in damped structures.

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